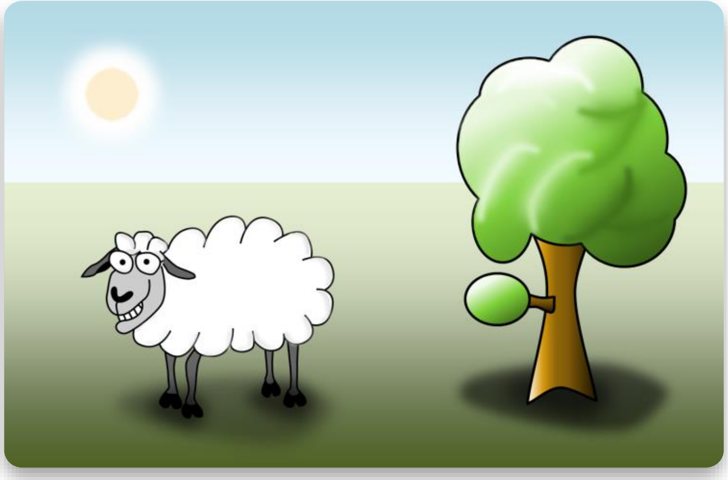
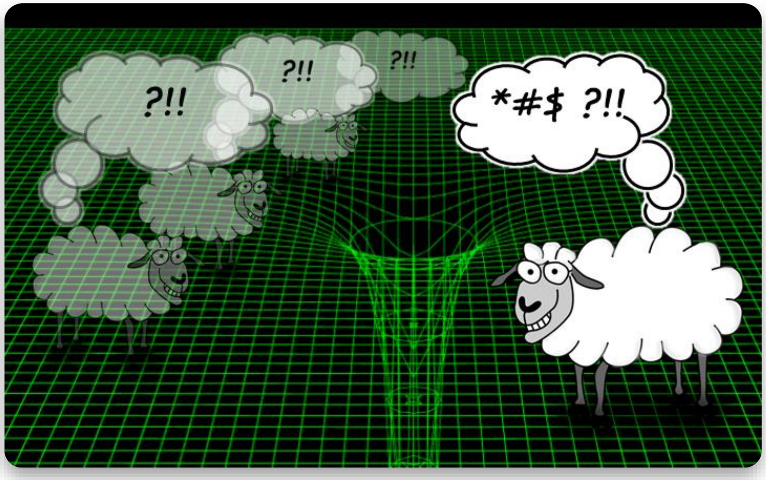
# Modelling 1

## SUMMER TERM 2020

Institut für Informatik

JOHANNES GUTENBERG UNIVERSITÄT MAINZ






## LECTURE 1

# Introduction

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# Course Overview

# Lecture Topic

## Modelling

- Natural phenomena
- Rebuild the outside world in the computer

## Two Problems

- Forward: Simulation (Model → Data)
- Backward: Inverse Problems (Data → Model)

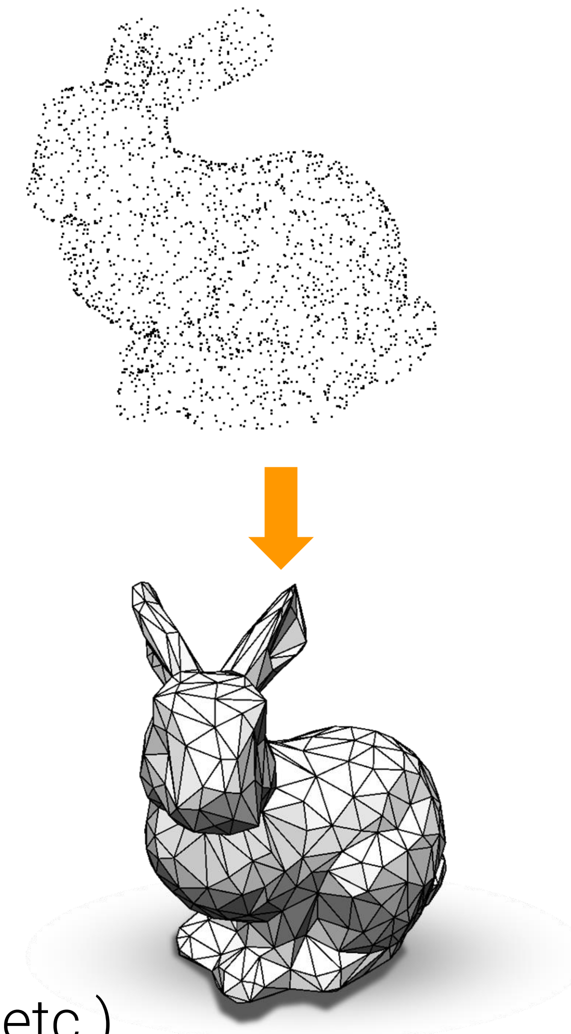
## Technical Approach

- Mathematical modeling + numerical algorithms
- Mostly (applied) linear algebra

# Lecture Content

## Topic

- Mathematical-numerical Modeling
- Application oriented
  - Focus on intuition
  - Less mathematical rigor / few proofs
  - „Getting things done“
- Examples focus on
  - Geometry
  - Dynamic systems
- Canonical examples
  - Reconstruction (from images, 3D scans etc.)



# Topics

## Modeling

- Modeling = Representation + Rules

## Representations

- Mathematical representations
  - What kind of / how much information is there in a system/phenomenon
- Digital representations
  - Discretization
- Tools / theory for analysis
  - What is going on here?

# The Answer to All Questions

**Spoiler**

- The answer will almost always be:

  Find a good coordinate system!

- (Because the questions will translate to

  What is the right coordinate system?)

# Topics

## Rules / Dynamics

- How does the system behave / evolve?
  - Space / time / both
  - Parameters?
- Modeling toolkit / examples
- Anecdotal / exemplary
  - More focus on representations

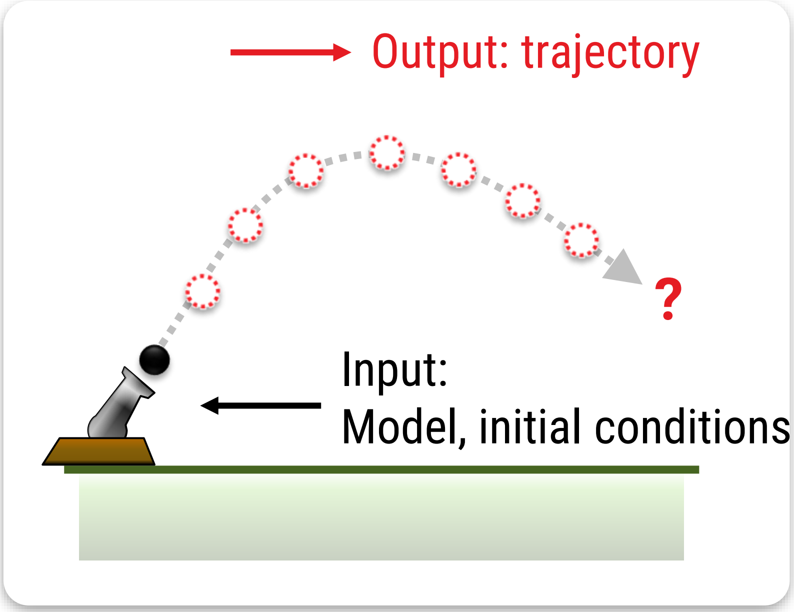
## Analysis

- Understand our model
- Understand the data

# Topic: Simulation

## Simulation

- „Forward“ simulation
- Predict system evolution



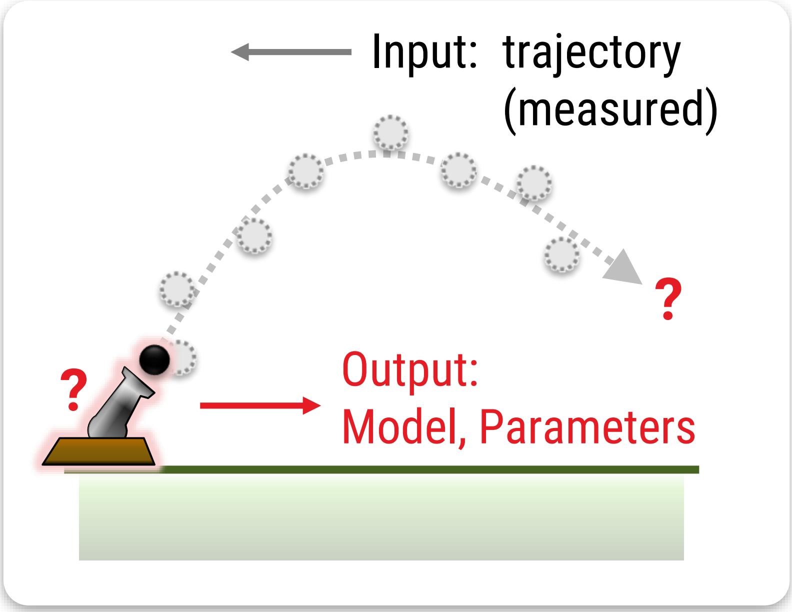

## Inverse problems

- Estimate reality from data
  - Noisy (measured) data given
  - Model (assumptions) given
  - Fit model parameter for optimal explanation of data
- Variational modeling
- “Ill-posed problems”

# Topics: Inverse Problems

## Simulation

- „Forward“ simulation
- Predict system evolution



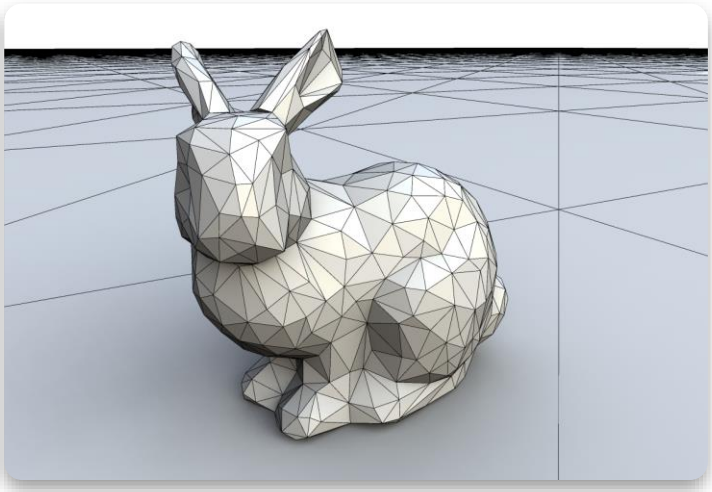

## Inverse problems

- Estimate reality from data
  - Noisy (measured) data given
  - Model (assumptions) given
  - Fit model parameter for optimal explanation of data
- Variational modeling
- “Ill-posed problems”

# Background

## Mathematical Modeling

- Tour through undergraduate engineering mathematics
- Connection to applications
- “Theoretical Computer Graphics”

# Mathematical Modeling

# Mathematical Models

## Mathematics

- Building *models*
- Understand their structure
- Computer science: generic/polymorphic algorithms
  - Think of "templates, super classes"

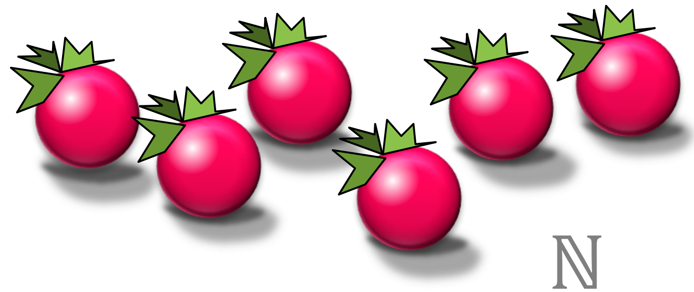
## How to understand structure?

- Make assumptions (axioms)
- Determine the consequences
- "If you assume {a, b, c} you will also have {x, y, z}"

# Example

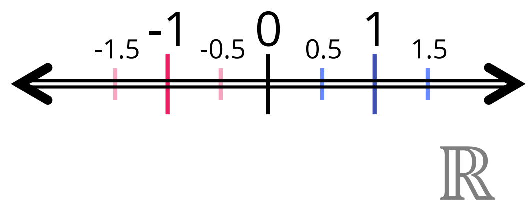
## Natural numbers

- 1,2,3,4,…
- Operations: +, −,*, /
- Model of discrete entities



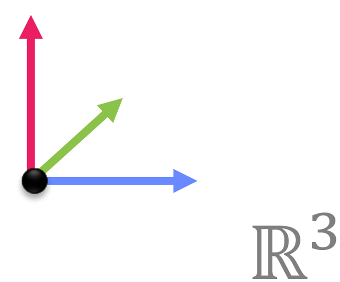

## Real numbers

- Model of a oriented line




## Real vector spaces

- Models of Euclidean spaces



# Three Aspects

**How to capture structure?**

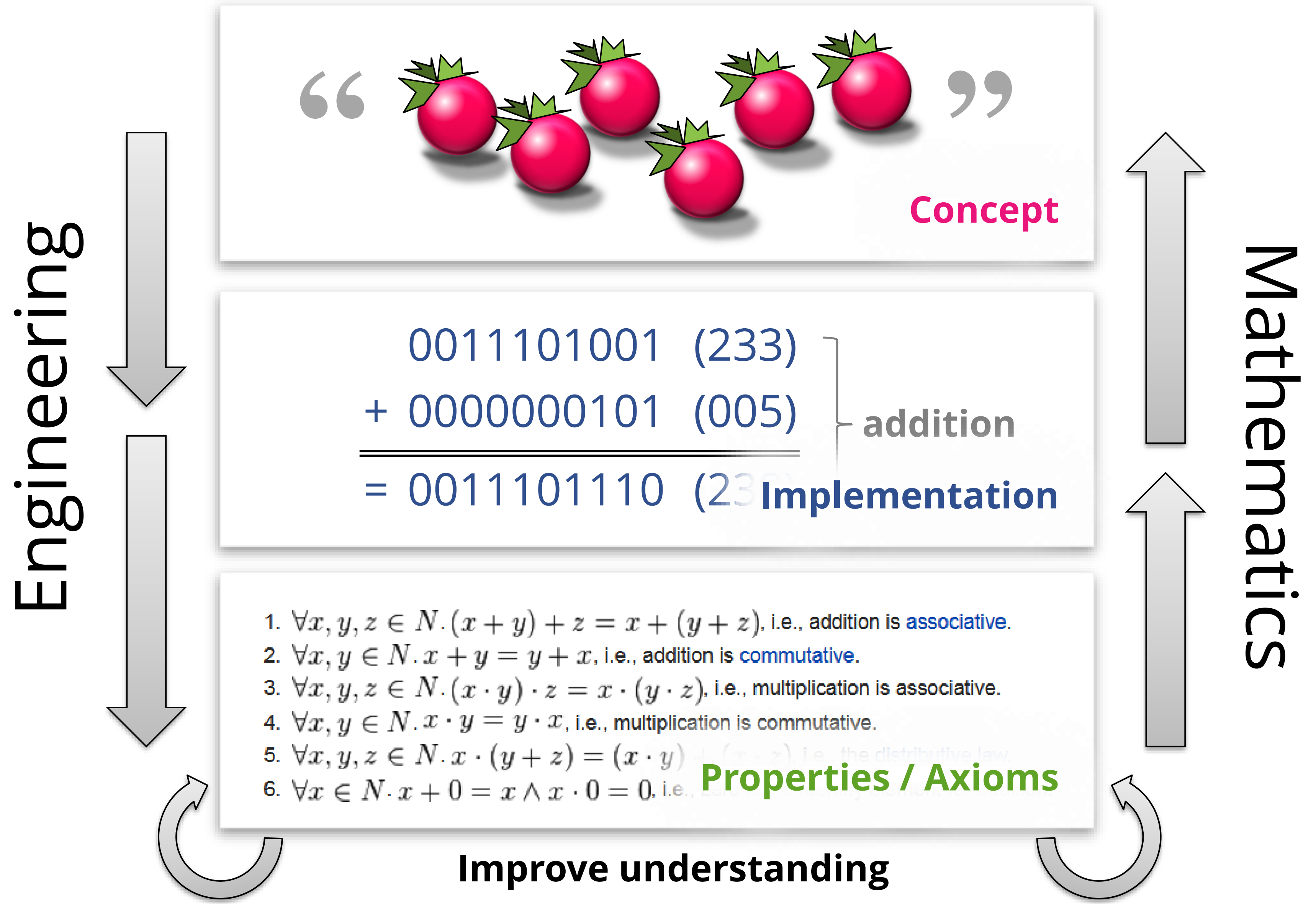
**Three components**

- Concept
- Implementation
- Properties

**Example**

- Natural numbers $\mathbb{N}$

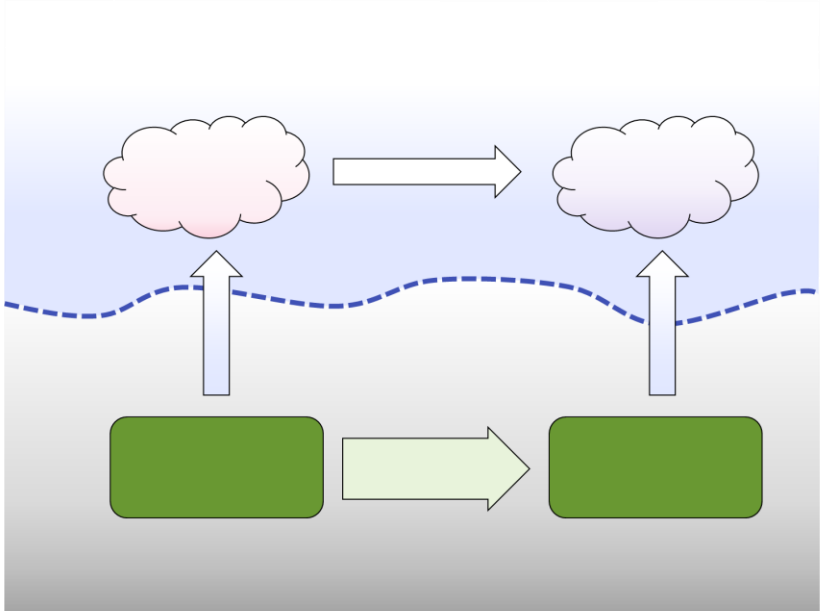
# Math vs. CS/Engineering



# Scientific Models

## Different subject areas

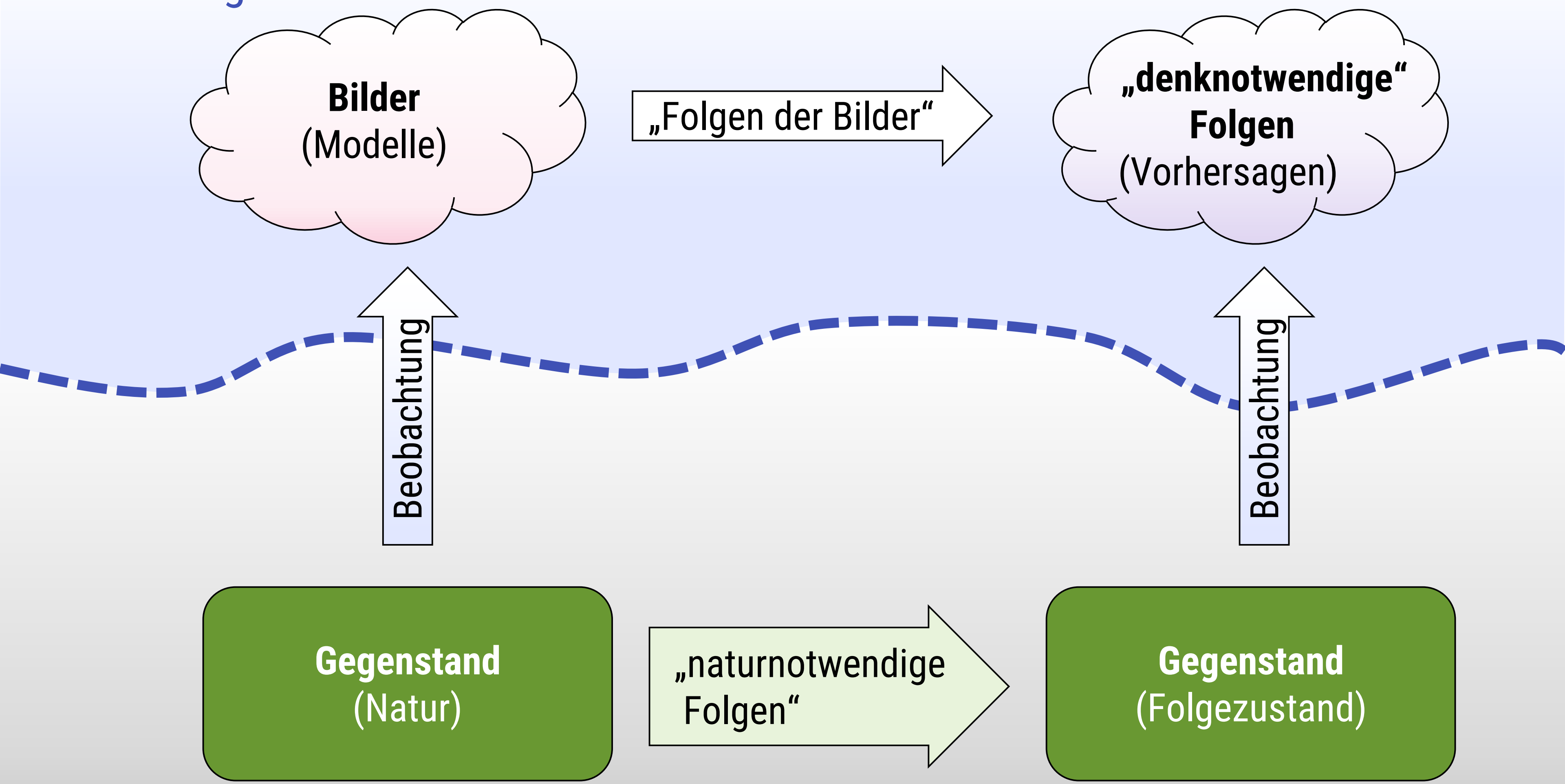
- Mathematical modeling
  - *Deduction*
  - Proofs for facts derived from axioms
- Empirical Modeling (physics, sociologie, biologie, …)
  - *Induction*
  - Find models that interpolate experimental data
  - Compact model that makes correct predictions
- (Applied) Computer Science
  - *Engineering*
  - Verify (inductive and/or deductive) that a system serves its purpose/goal ("best practices")



# Model Induction

# Prinzip (H. Hertz, 1894)

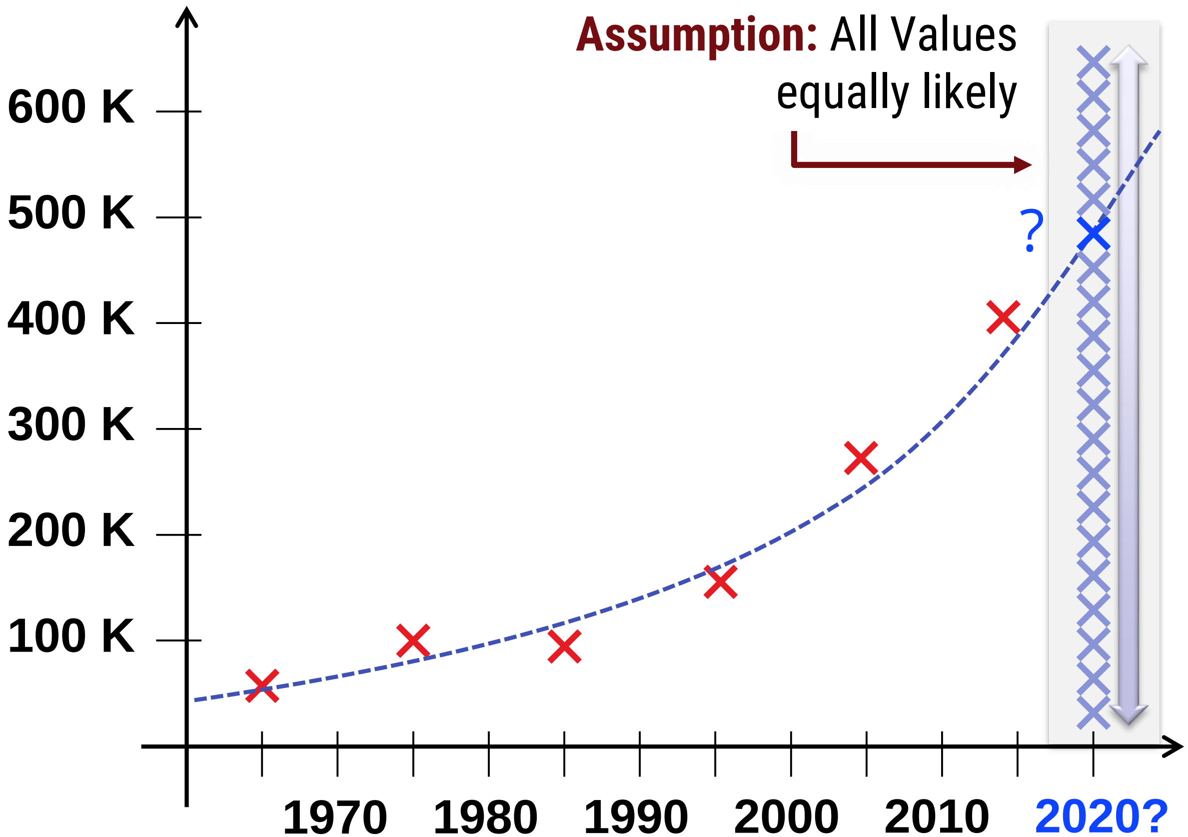
**Ideelle Welt**
Modelle,
Vorstellungen




**Reale, objektive Welt (unbekannt)**

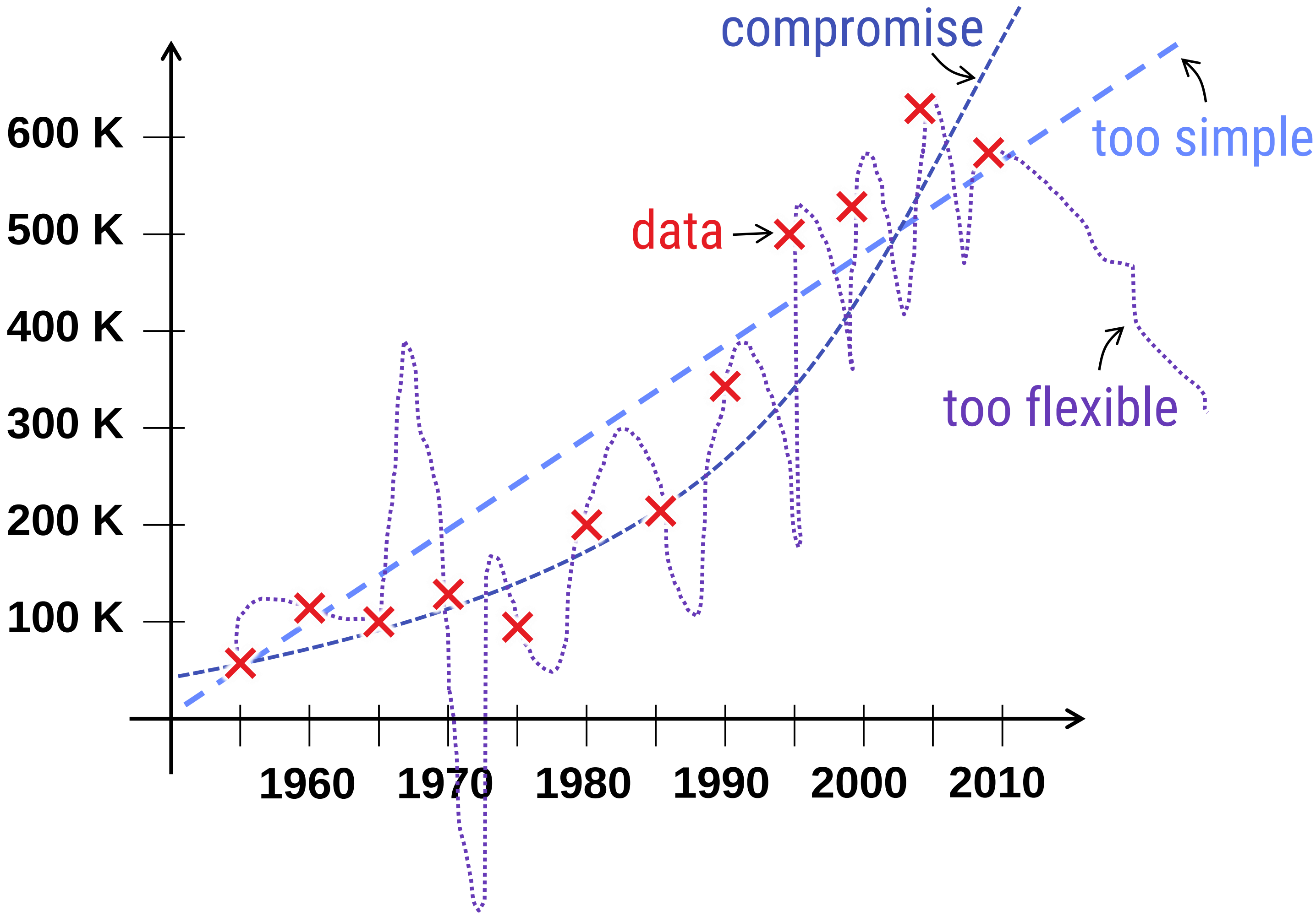
# Making Predictions

## Predicting the housing market

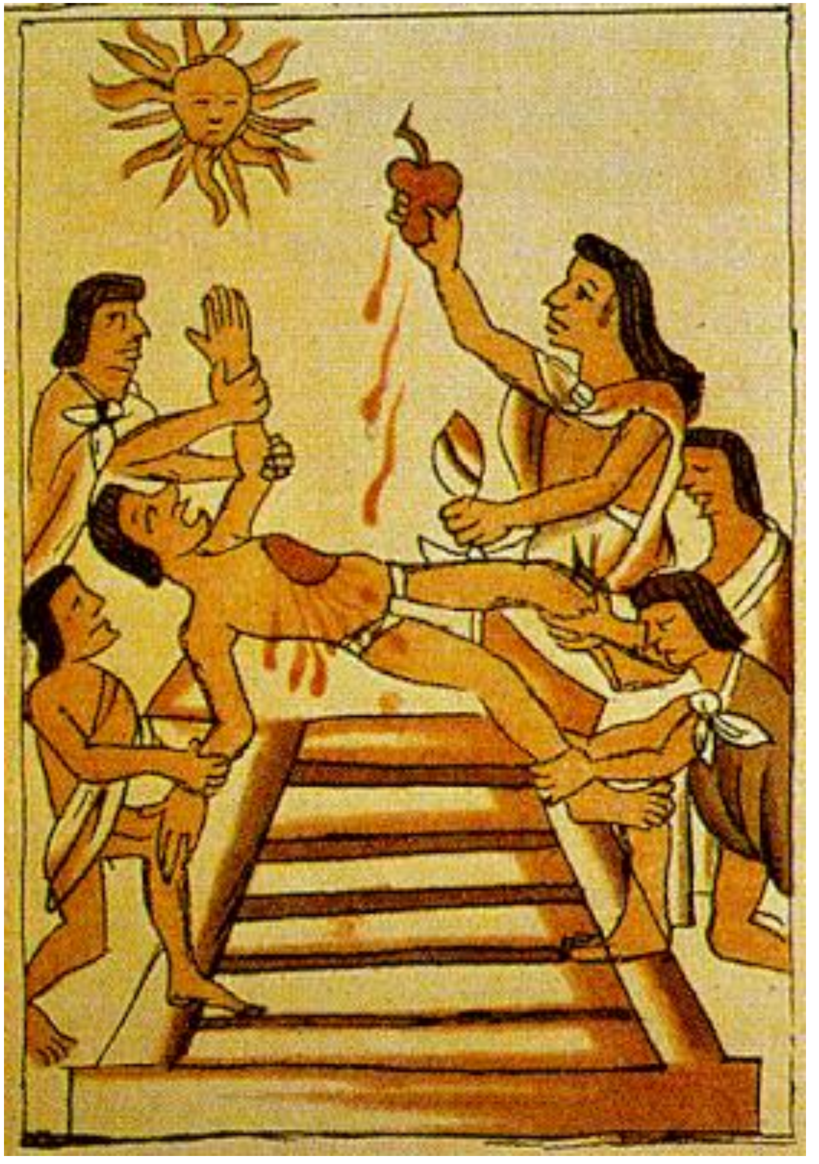



disclaimer: numbers are made up
this is no investment advice

# Making Predictions



# “Overfitting”



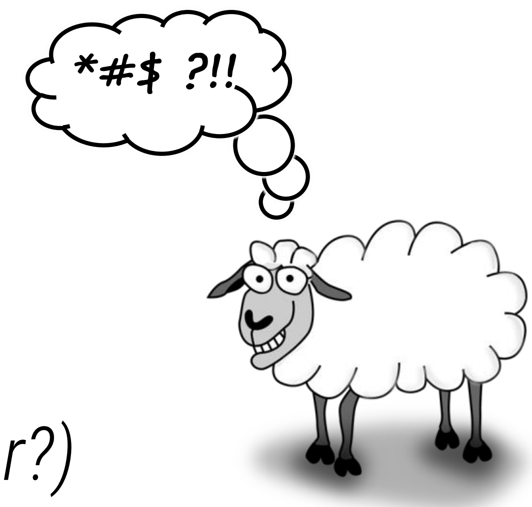
# Good Models (Theories)

## Requirements for a good Model/Theory

- **H. Hertz:** Theory must make correct predictions
  - Up to measurement accuracy, of course
- **K. Popper:** Theory must be falsifiable
  - Model parameters must affect predictions in a measurable way
- **W. Occam:** Among all explanations that predict correctly, choose the simplest one
  - "Occam's Razor"

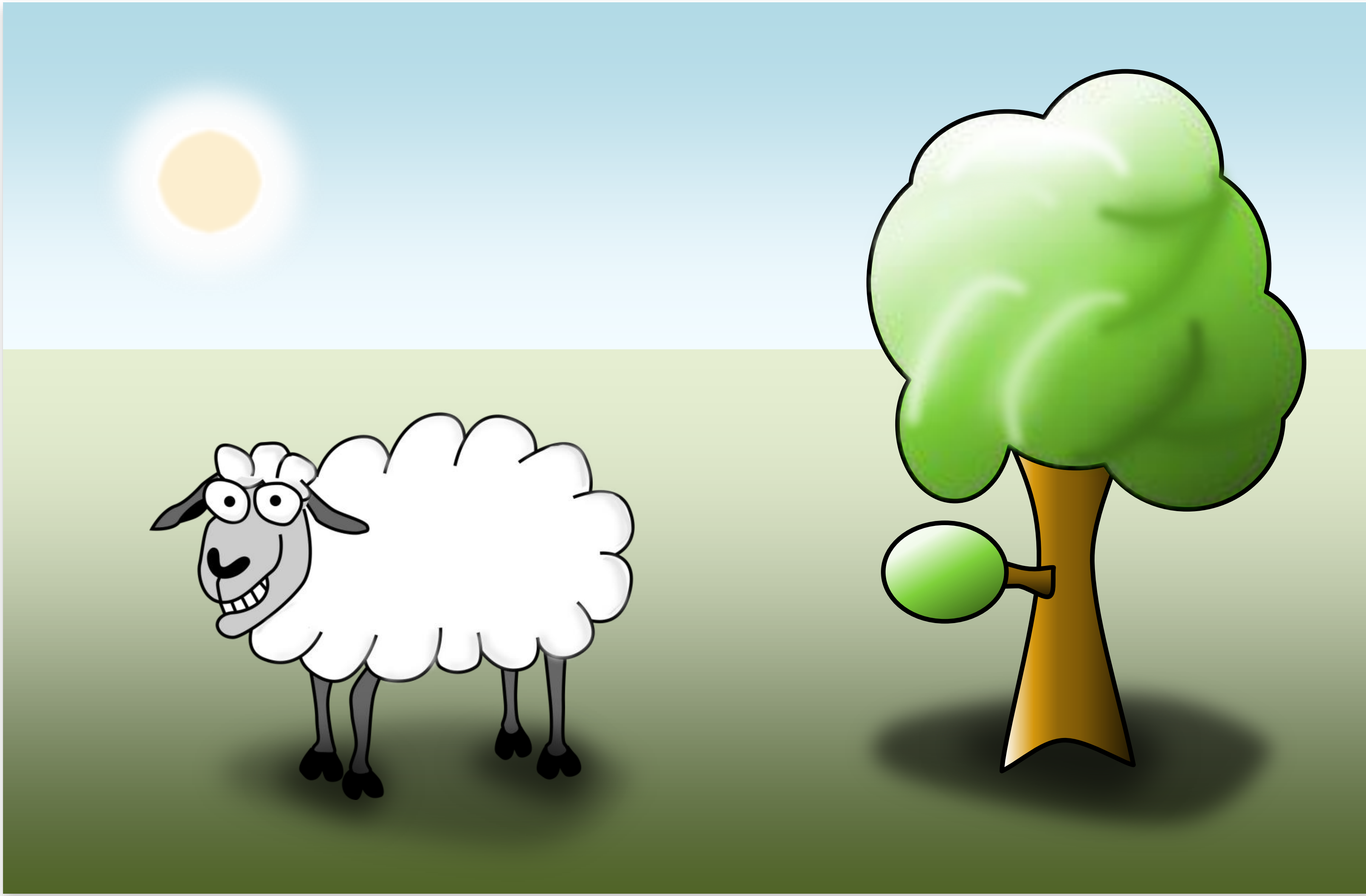
## Hypothesis:

# Evolution of Mathematial Models




*(Why does all the theory seem so linear?)*

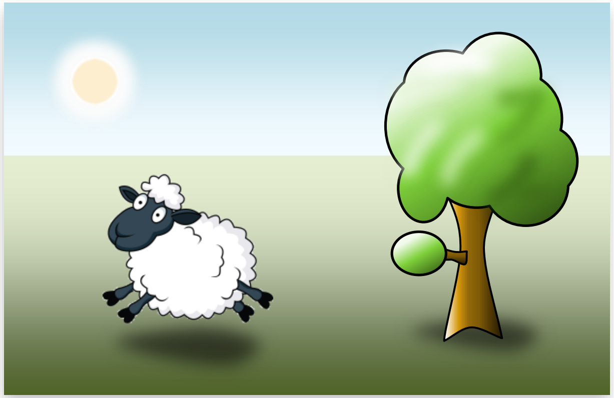
# Evolution of Mathematical Modeling



# Evolution of Mathematical Modeling

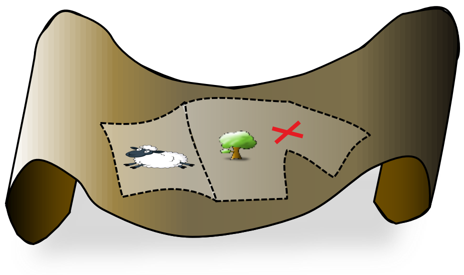
## Application area

- Agriculture
- Sheep & wheat



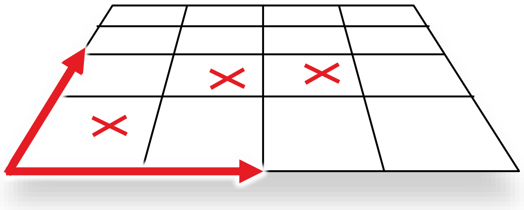
## Engineering

- Surveying



## Mathematical abstraction

- Euclidean geometry
- Vector spaces



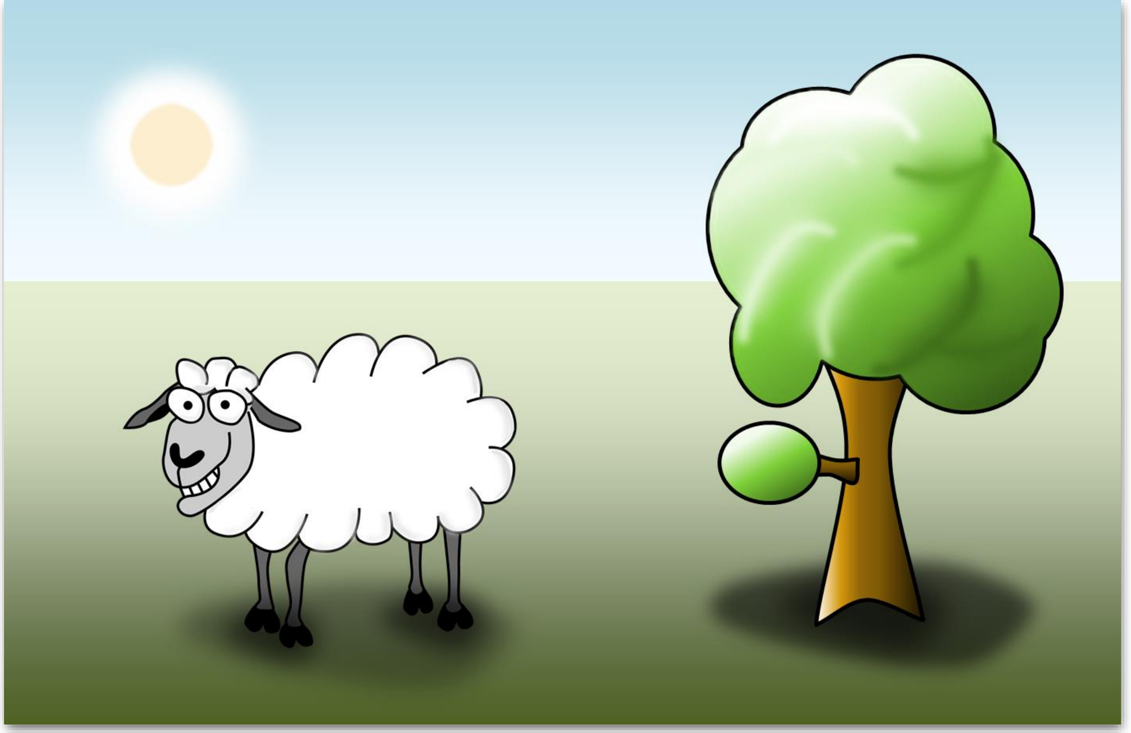
# Evolution of Modeling

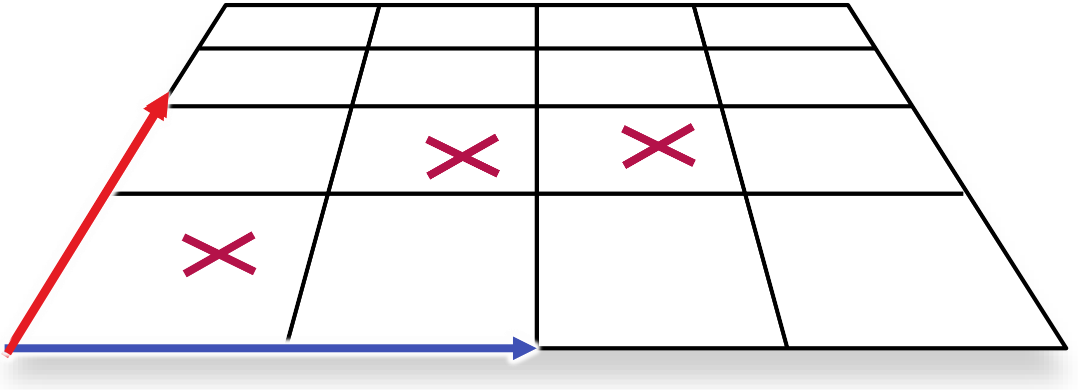
**Richard Dawkins:**

*Why the Universe seems so strange*, Ted Talk, 2005.

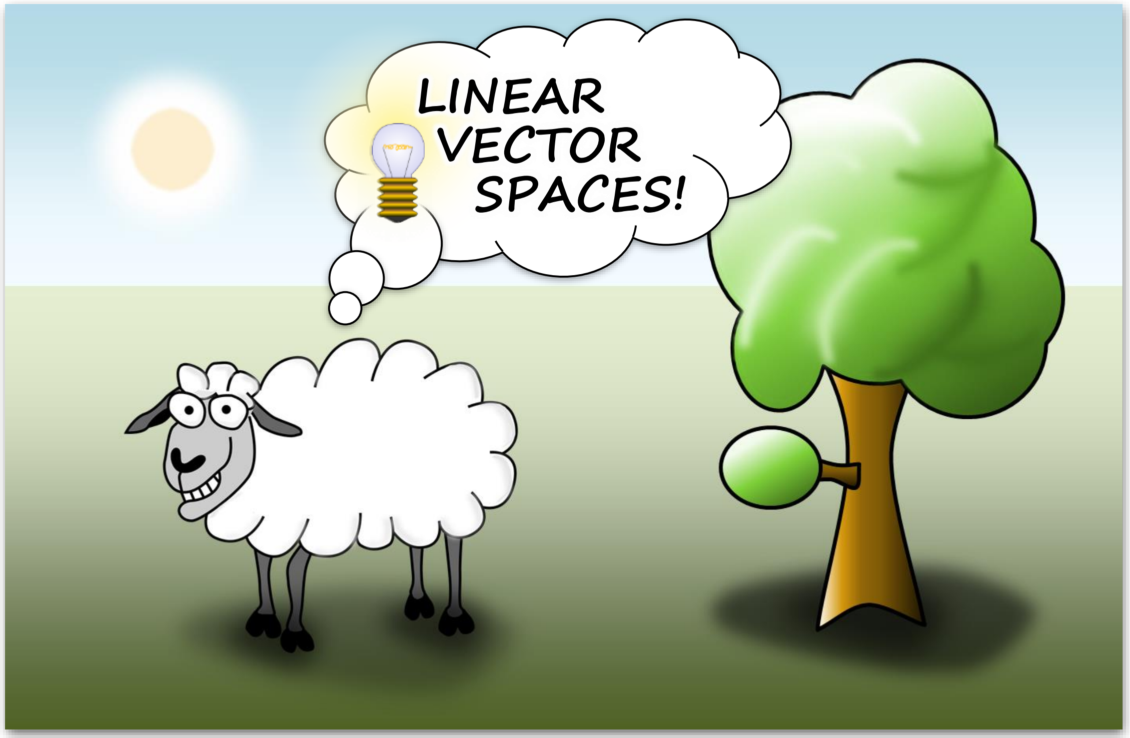
Moving to the other end of the scale, our ancestors never had to navigate through the cosmos at speeds close to the speed of light. If they had, our brains would be much better at understanding Einstein. I want to give the name "Middle World" to the medium-scaled environment in which we've evolved the ability to take act -- nothing to do with Middle Earth. Middle World. (Laughter) We are evolved denizens of Middle World, and that limits what we are capable of imagining. We find it intuitively easy to grasp ideas like, when a rabbit moves at the sort of medium velocity at which rabbits and other Middle World objects move, and hits another Middle World object, like a rock, it knocks itself out.

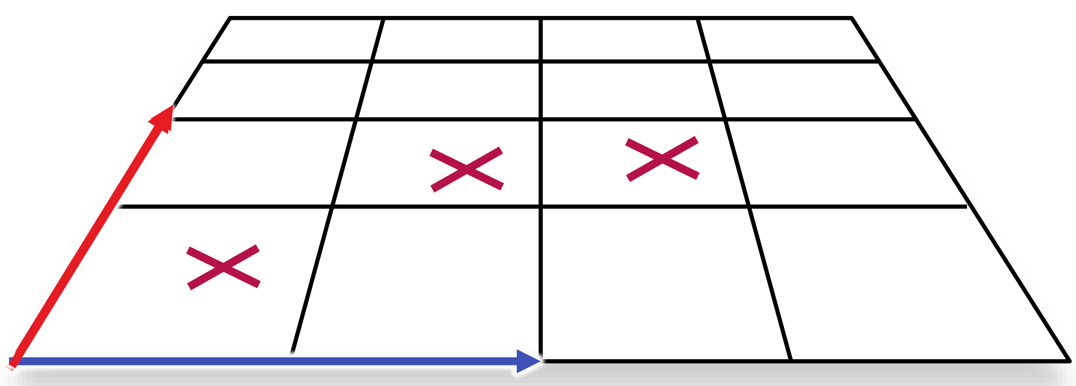
# Evolution of Mathematical Modeling





# Evolution of Mathematical Modeling



# Questions for Linear Modeling

## Modeling I

- Linear/Euclidean spaces
- High-dimensional spaces

→ **LINEAR ALGEBRA**

## Questions

- How to represent phenomena?
  - Select a good linear space.
- How implicitly characterize them?
  - Functional equations, variational models
- What information is in a linear model?
  - Store & process efficiently in a computer

# How to go beyond the wide, flat green…

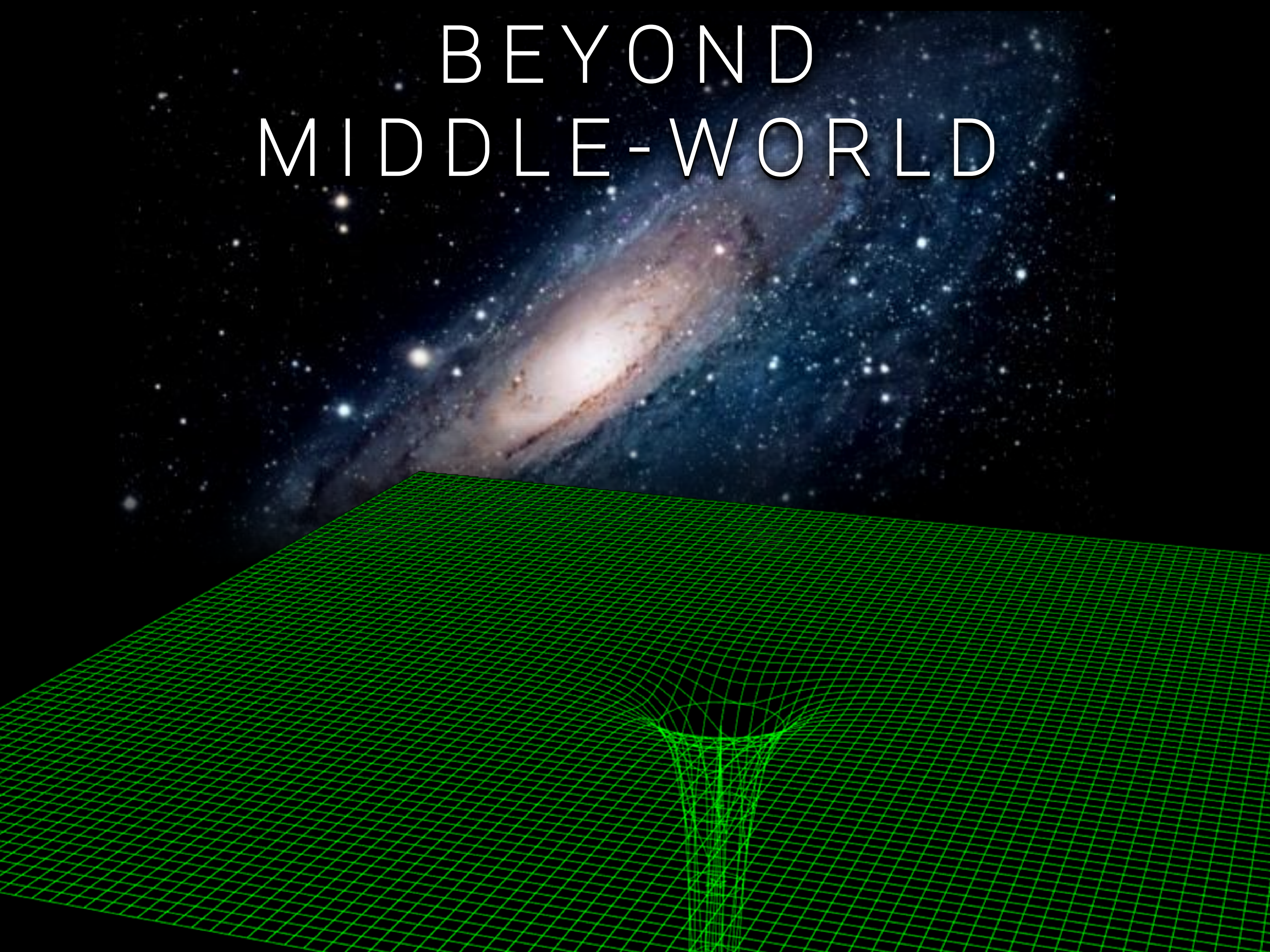
## Modeling of Curved…

- Things
  - Objects, Trajectories, etc.
  - Polynomials
  - Differential equations
- Spaces
  - Space itself
  - Differential geometry

→ **CALCULUS**

## **Common Recipe:** Taylor-Approximation

- Linearly describe deviation from linear, recursively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